A Polarized Spacetime Foam

V. Dzhunushaliev*
Dept. Phys. and Microel. Engineer., Kyrgyz-Russian Slavic University, Bishkek, Kievskaya Str. 44, 720000, Kyrgyz Republic
(Dated: January 30, 2002)

An approximate model of a spacetime foam is presented. It is supposed that in the spacetime foam each quantum handle is like to an electric dipole and therefore the spacetime foam is similar to a dielectric. If we neglect of the linear sizes of the quantum handle then it can be described with some operator containing a Grassman number and either a scalar or a spinor field. For both fields the Lagrangian is presented. For the scalar field it is the dilaton gravity + electrodynamics and the dilaton field is a dielectric permeability. The spherically symmetric solution in this case give us the renormalization of a bare electric charge surrounded by a polarized spacetime foam and the energy of the electric field become finite one. In the case of the spinor field the spherically symmetric solution give us a ball of the polarized spacetime foam filled with a electric field. The full energy of the electric field in the ball can be compared with the energy of the Gamma Ray Burst.

arXiv:gr-qc/0109050 14 Sep 2001

I. INTRODUCTION

One of the manifestation of quantum gravity is a spacetime foam introduced by Wheeler [1] for the description a hypothesized topology fluctuation on the Planck scale level. The spacetime foam is a cloud of appearing/disappearing quantum handles. The appearance/desctruction of these handles leads to the change of spacetime topology. This fact give rise to big difficulties at the description of the spacetime foam since by the topology changes of a space (according to Morse theory) the critical points must exist where the time direction is not defined. In each such point should be a singularity which is an obstacle for the mathematical description of the spacetime foam.

Nevertheless we can try to describe spacetime foam by some approximate manner. For the beginning we offer a model of a quantum handle as the wormhole-like metric in the Kaluza-Klein gravity. In some approximation we can neglect the linear sizes of the handle and in this case each quantum handle looks as two points pasted together. It is so called minimalist wormhole [2], *i.e.* quantum wormhole in which the cross section of the throat is contracted to a point. Afterwards we will introduce an operator which describes the creation/annihilation of the minimalist wormhole. We will show that this operator can be presented as a Grassman number and some (scalar or fermion) field.

One mouth of such quantum wormhole can entrap the electric force lines then these force lines will emerge from another mouth. It allows us to consider each quantum wormhole as an electric dipole and the spacetime foam as a dielectric [3]. In this case a big electric field can polarize the spacetime foam that can be the reason for some physical effects.

II. THE MODEL OF A QUANTUM HANDLE

The model of the individual QWH is presented on the Fig.(1). It is some realization of the Wheeler idea about a wormhole entrapping electric force lines. In Ref.[1] he wrote: "Along with the fluctuations in the metric there occur fluctuations in the electromagnetic field. In consequence the typical multiply connected space ... has a net flux of electric lines of force passing through the "wormhole". These lines are trapped by the topology of the space. These lines give the appearance of a positive charge at one end of the wormhole and a negative charge at the other". The composite wormhole on the Fig.(1) consists from two Reissner-Nordström black holes and the 5D throat inserted between them [4]. The 5D metric for this throat is

$$ds^2_{(5)} = -R_0^2 e^{2\psi(r)}\Delta(r)\left(d\chi + \omega(r)dt + Q\cos\theta d\varphi\right)^2 + \frac{1}{\Delta(r)}dt^2 - dr^2 - a(r)\left(d\theta^2 + \sin\theta^2 d\varphi^2\right), \tag{1}$$

where χ is the 5^{th} extra coordinate; R_0 and Q are some constants. We assume that in some rough approximation the quantum handle in the spacetime foam can be presented by this manner. The 5D Einstein equations for the throat

*Electronic address: dzhun@hotmail.kg

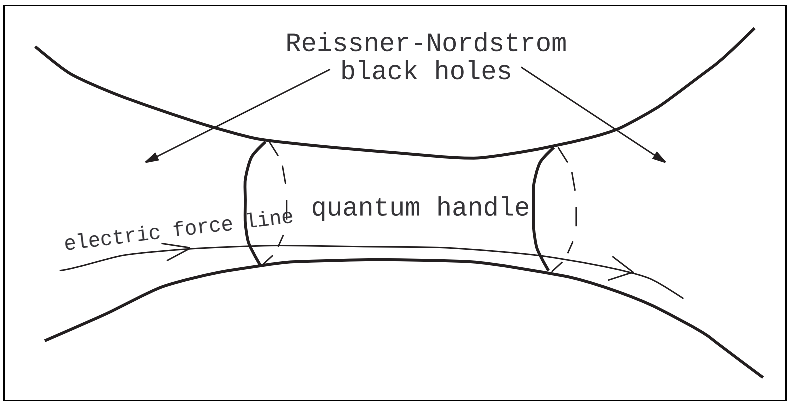

FIG. 1: The model of a quantum handle. The whole spacetime is 5 dimensional and: in the Reissner-Nordström black hole $G_{55} = const$ and it is not varying (this is the 5D gravity in the initial Kaluza-Klein interpretation); in the 5D throat G_{55} is the dynamical field.

are

$$\frac{\Delta''}{\Delta} - \frac{{\Delta'}^2}{\Delta^2} + \frac{\Delta'\psi'}{\Delta} + \frac{a'\Delta'}{a\Delta} + R_0^2{\omega'}^2\Delta^2 e^{2\psi} = 0, \tag{2}$$

$$\omega'' + \omega'\left(2\frac{\Delta'}{\Delta} + 3\psi' + \frac{a'}{a}\right) = 0, \tag{3}$$

$$\frac{a''}{a} + \frac{a'\psi'}{a} - \frac{2}{a} + \frac{Q^2\Delta e^{2\psi}}{a^2} = 0, \tag{4}$$

$$\psi'' + {\psi'}^2 + \frac{a'\psi'}{a} - \frac{Q^2\Delta e^{2\psi}}{2a^2} = 0, \tag{5}$$

$$\frac{{\Delta'}^2}{\Delta^2} + 2\frac{\Delta'\psi'}{\Delta} - 4\frac{a'\psi'}{a} + \frac{4}{a} - \frac{{a'}^2}{a^2} - R_0^2{\omega'}^2\Delta^2 e^{2\psi} - \frac{Q^2\Delta e^{2\psi}}{a^2} = 0. \tag{6}$$

It can be shown [5] that there is three type of solutions : the first type (wormhole-like solution) is presented on Fig.(1) with $E > H$ (E and H are Kaluza-Klein electric and magnetic fields), the second one is an infinite flux tube with $E = H$ and the third one is a singular solution (finite flux tube) with $E < H$. The longitudinal size L_0 of the WH-like solution depends on the relation between electric and magnetic fields : if $(1 - H/E) \to 0$ then $L_0 \to \infty$. Let us define an approximate solution close to points $r^2 = r_0^2$ (where $ds^2(\pm r_0) = 0$).

$$\Delta(r) \approx \Delta_1\left(r_0^2 - r^2\right), \tag{7}$$

$$\omega(r) \approx \frac{\omega_1}{r_0^2 - r^2}, \tag{8}$$

$$\psi(r) \approx \frac{\psi_3}{6}\left(r_0^2 - r^2\right)^3. \tag{9}$$

with

$$\Delta_1 = \pm\frac{q}{2a_0 r_0}, \tag{10}$$

$$\omega_1 = \frac{2a_0 r_0}{q}, \tag{11}$$

$$\psi_3 = \pm\frac{qQ^2}{2a_0^3 r_0^3} \tag{12}$$

here $a_0 = a(r = \pm r_0)$, q is some constant. It is easy to show that at the hypersurfaces $r = \pm r_0$: $ds^2 = 0$. On these hypersurfaces the change of the metric signature takes place : $(+,-,-,-,-)$ by $|r| < r_0$ and $(-,-,-,-,+)$ by $|r| > r_0$. Following to Bronnikov [6] we call these two hypersurfaces as $T-$horizons.

For the definition of a Kaluza-Klein electric field we consider Eq.(3)

$$\left[\left(\omega' \Delta^2 e^{3\psi}\right) 4\pi a\right]' = 0 \tag{13}$$

here $4\pi a$ is the area of S^2 sphere. Comparing with the Gauss law we see that Kaluza-Klein electric field E_{KK} can be defined as follows

$$E_{KK} = \omega' \Delta^2 e^{3\psi} = \frac{q}{a} \tag{14}$$

here q is an electric charge which is proportional to a flux of electric field. In this case the force lines of the electric field are uninterrupted and can be continued through the surfaces of matching the 5D WH-like solution and the Reissner-Nordström solution like to Fig.1.

On these $T-$horizons we should match:

- the flux of the 4D electric field (defined by the Maxwell equations) with the flux of the 5D electric field defined by $R_{5t} = 0$ Kaluza-Klein equation.
- the area of the Reissner-Nordström event horizon with the area of the $T-$horizon.

In the next section we will consider an approximate model of the spacetime foam where we neglect the linear sizes of the quantum handle. It means that the cross section and longitudinal sizes of the throat are contracted into a point. Such approximation for the quantum handle is called as a minimalist wormhole. And further we will consider the consequences for such approximation.

III. THE OPERATOR DESCRIPTION OF A MINIMALIST WORMHOLE

Let us introduce an operator $W(x,y)$ which describes creation/annihilation minimalist wormhole connected two points x and y [2] [7]. Let the operator $W(x,y)$ has the following property

$$W^2(x,y) = 0. \tag{15}$$

It means that the reiterated creation/annihilation minimalist wormhole is senseless. This property tell us that the operator $W(x,y)$ can be connected with the Grassman numbers. In the general case the operator $W(x,y)$ is nonlocal one but in this paper we will consider the case when $W(x,y)$ can be factorized on two local operators

$$W(x,y) = \theta(x)\theta(y). \tag{16}$$

The operator $\theta(x)$ can be considered as a readiness of a point x to pasting together with a point y (or conversely the separation these points in the minimalist wormhole).

We will consider three cases.

The first case is

$$W^{ab}(x,y) = \phi(x)\theta^a\phi(y)\theta^b \tag{17}$$

where a, b are the undotted spinor indices, $\phi(x)$ is the scalar field, θ^a is the Grassman number

$$\theta^a\theta^b + \theta^b\theta^a = 0. \tag{18}$$

It is easy to proof that

$$W^{ab}(x,y)W_{ab}(x,y) = \phi^2(x)\phi^2(y)\theta^a\theta^b\theta_a\theta_b = \phi^2(x)\phi^2(y)\varepsilon_{aa'}\varepsilon_{bb'}\theta^a\theta^b\theta^{a'}\theta^{b'} \equiv 0 \tag{19}$$

where

$$\varepsilon^{ab} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad \varepsilon_{ab} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \tag{20}$$

Eq. 19 means that it makes no sense to identify two points twice.

The second case is

$$W(x,y) = \psi_a(x)\theta^a\psi_b(y)\theta^b = \psi_a(x)\psi_b(y)\theta^a\theta^b = \left(\psi_1(x)\psi_2(y) - \psi_2(x)\psi_1(y)\right)\theta^1\theta^2 \tag{21}$$

where $\psi_a(x)$ is an undotted spinor field in $(\frac{1}{2},0)$ representation. The square is zero

$$W^2(x,y) = \left(\psi_1(x)\psi_2(y) - \psi_2(x)\psi_1(y)\right)^2 \theta^1\theta^2\theta^1\theta^2 \equiv 0. \tag{22}$$

In this case $W(x,x) \equiv 0$. It means that the minimalist wormhole can connect only two different points.

The third case is

$$W(x,y) = \psi_a(x)\theta^a \psi_{\dot b}(y)\bar\theta^{\dot b} = \psi_a(x)\psi_{\dot b}(y)\theta^a\bar\theta^{\dot b} \tag{23}$$

where $\psi_{\dot b}$ is a dotted spinor in $(0,\frac{1}{2})$ representation. The square is zero

$$W^2(x,y) = \left(\psi_a(x)\psi_{\dot b}(y)\theta^a\bar\theta^{\dot b}\right)\left(\psi_c(x)\psi_{\dot d}(y)\theta^c\bar\theta^{\dot d}\right) = \left(\psi_a(x)\psi_c(x)\theta^a\theta^c\right)\left(\psi_{\dot b}(y)\psi_{\dot d}(y)\bar\theta^{\dot b}\bar\theta^{\dot d}\right) \equiv 0 \tag{24}$$

as

$$\psi_a(x)\psi_c(x)\theta^a\theta^c = \psi_1(x)\psi_2(x)\theta^1\theta^2 + \psi_2(x)\psi_1(x)\theta^2\theta^1 \equiv 0. \tag{25}$$

We must note that as the classical canonical theory cannot describes topology change these operators do not correspond to any classical observables. It corresponds to the well-known fact that the Grassman numbers do not have any classical interpretation.

We see that $W(x,y)$ operator is described with dynamical fields : scalar field $\phi(x)$ or spinor field $\psi(x)$. Below we will present some equations for these fields and discuss the physical sense of corresponding solutions.

IV. THE SCALAR FIELD

For the description of the dynamics of the scalar field we offer the following (dilaton gravity + Maxwell) Lagrangian

$$\mathcal{L} = -\sqrt{-g}\left(R + 2\nabla_\mu\phi\nabla^\mu\phi - e^{-2\phi}F_{\mu\nu}F^{\mu\nu}\right) \tag{26}$$

where g is the determinant of the 4D metric, R is the Ricci scalar, ϕ is the dilaton field and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_n u A_\mu$ is the tensor of the electromagnetic field, $\mu = 0,1,2,3$. It means that we interpret the above-mentioned scalar field as the dilaton field. The corresponding field equations are

$$\nabla_\nu\left(e^{-2\phi}F^{\mu\nu}\right) = 0, \tag{27}$$

$$\nabla_\mu\nabla^\mu\phi = \frac{1}{2}e^{-2\phi}F_{\mu\nu}F^{\mu\nu}, \tag{28}$$

$$R_{\mu\nu} - \frac{1}{2}R = \varkappa T_{\mu\nu} \tag{29}$$

where $R_{\mu\nu}$ is the Ricci tensor and $T_{\mu\nu}$ is the tensor-momentum for the dilaton and Maxwell fields. Each minimalist wormhole (as a quantum handle in the spacetime foam) looks as a dipole because one mouth entraps the electric force lines and another mouth will emerges these force lines. It allows us to consider $e^{-2\phi}$ as a dielectric permeability

$$\varepsilon(r) = e^{-2\phi(r)}. \tag{30}$$

In this case we can introduce the electric displacement $\vec D$

$$\vec D = \varepsilon\vec E = e^{-2\phi}\vec E = \vec E + \vec P = (1 + 4\pi\chi)\vec E \tag{31}$$

where $\vec P$ is the polarization vector.

We would like to consider the static spherically symmetric case with the metric

$$ds^2 = A^2(r)dt^2 - \frac{dr^2}{A^2(r)} - \rho^2(r)\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{32}$$

$$F_{tr} = E_r \neq 0. \tag{33}$$

The solution is [8]

$$A^2(r) = 1 - \frac{r_+}{r}, \tag{34}$$

$$\rho^2(r) = r(r-2r_0), \tag{35}$$

$$e^{2\phi(r)} = 1 - \frac{2r_0}{r}, \tag{36}$$

$$E_r(r) = \frac{Q}{r^2}, \tag{37}$$

$$D_r(r) = \frac{Q}{\rho^2}, \tag{38}$$

$$r_0 = \frac{Q}{r_+} \tag{39}$$

where r_+ and r_0 are some constants, Q is the electric charge. For us interesting is the following case

$$r_0 \gg r_+ \qquad \text{or} \qquad r_+ \ll Q \tag{40}$$

where the electric charge and field is in the natural units : $[Q] = cm$ and $[E] = cm^{-1}$. It is necessary to note that we have a gravitational singularity at the point $r = 2r_0$ ($\rho = 0$).

Our interpretation of this solution is the following. It is a bare electric charge Q surrounded by a polarized spacetime foam (PSF). The PSF is described by the dilaton field ϕ which is connected with the operator $W(x,y)$ describing the minimalist wormhole (17). At the infinity $\phi(r) = 0$ that means $W(x,y) = 0$ and $\varepsilon = e^{-2\phi} = 1$, *i.e.* the PSF absents at the infinity.

Now we would like to present some physical consequences of our model of the bare charge imbedded in the PSF.

A. The renormalization of a bare charge

From Eq. (31) we have

$$4\pi P_r = D_r - E_r = Q\left(\frac{1}{\rho^2} - \frac{1}{r^2}\right). \tag{41}$$

The surface density σ of the bound charges is

$$\sigma = P_n = P_r \tag{42}$$

where P_n is the normal component of the $\vec{P}$. Consequently a charge Q_{anti} on the little sphere surrounding a bare charge Q is

$$Q_{anti} = -\left|4\pi\rho^2\sigma\right| = -Q\left(1 - \frac{\rho^2}{r^2}\right) = -Q\frac{2r_0}{r} \xrightarrow{r\to 2r_0} -Q. \tag{43}$$

Thus the PSF completely shield the bare electric charge. Another consequence is that the PSF intercepts all the force lines of the bare charge and redistributes their in the surrounding space.

B. The energy of the electric charge

Let us calculate the energy W of the electric field

$$W = \frac{1}{8\pi}\int_{2r_0}^{\infty} D_r E_r \sqrt{-\gamma}dr \approx \frac{Q^2}{4r_0} \tag{44}$$

where we have used the condition $r_+ \ll r_0$ and γ is the determinant of the 3D metric. It is interesting compare this energy with the rest energy of electron

$$m_0c^2 = \frac{e^2}{4r_0}, \qquad Q = e \tag{45}$$

where m_0 is the rest mass of electron. In this case $r_0 \approx 10^{-13}cm$ is the classical radius of electron and $r_+ \approx 10^{-45}cm$.

C. The gravitational effects

The metric approximately is

$$ds^2 \approx dt^2 - dr^2 - r(r - 2r_0)\left(d\theta^2 + \sin^2\theta d\varphi^2\right). \tag{46}$$

We see that the metric in this model is not flat on the level of the classical electron radius. The difference can be measured as a deficit of the sphere area

$$\delta = 1 - \frac{4\pi l_\rho^2}{4\pi\rho^2} \tag{47}$$

where l_ρ is the distance between the origin $(r = 2r_0)$ and the sphere with the radius $\rho = \sqrt{r(r - 2r_0)}$

$$l_\rho = \int_{2r_0}^{r} \frac{dr}{\sqrt{1 - \frac{r_+}{r}}} \approx (r - 2r_0). \tag{48}$$

It means that

$$\delta \approx 1 - \frac{(r - 2r_0)^2}{\rho^2} = \frac{2}{3} \qquad \text{by} \qquad r = 3r_0. \tag{49}$$

Let us remind that in the chosen coordinate system $r > 2r_0$.

Thus this model of electron with the PSF give us the finite rest energy of electron (connected with its renormalized electric field) but at the center of electron we have a gravitational singularity. Also we should note that at the center of electron the electric field $E_r = \frac{Q}{2r_0}$ is finite but the direction of this vector is undefined. The picture of $\vec{E}$ at the center is like to a hedgehog.

The experimental verified consequences of this model is the nonflat metric (46) and the difference between D_r and E_r. For example, at the distance $r - 2r_0 \approx r_0$ we have the deficit of the sphere area

$$\delta \approx \frac{2}{3} \tag{50}$$

and the dielectric permeability

$$\varepsilon = \frac{D_r}{E_r} = \frac{r^2}{\rho^2} = \frac{1}{1 - \frac{2r_0}{r}} \approx 3. \tag{51}$$

Evidently it is too much but it can be connected with that our dilaton model of the PSF is approximate one. Nevertheless on the basis of such model we have the interesting model of the renormalization of the bare electric charge with the finite energy of the electric field.

V. THE SPINOR FIELD

In this section we would like to consider the spinor model of the PSF with the following Lagrangian

$$\begin{aligned}\mathcal{L} \;=\; & \sqrt{-g}\left\{-\frac{1}{2k}\left(R + \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta}\right) + \right. \\ & \left.\frac{\hbar c}{2}\left[i\bar{\psi}\left(\gamma^\mu\nabla_\mu - \frac{1}{8}F_{\alpha\beta}\gamma^{[\alpha}\gamma^{\beta]} - \frac{1}{4}l_0^2\left(\gamma^5\gamma^\mu\right)\left(i\bar{\psi}\gamma^5\gamma_\mu\psi\right) - \frac{m}{i}\right)\psi + h.c.\right]\right\}\end{aligned} \tag{52}$$

where α, β, μ are the 4D world indexes, γ^μ are the 4D γ matrixes with usual definitions $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu}$, $\eta^{\mu\nu} = (+,-,-,-)$ is the signature of the 4D metric. Varying with respect to $g_{\mu\nu}$, $\bar{\psi}$ and A_μ leads to the following equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \varkappa T_{\mu\nu}, \tag{53}$$

$$D_\nu H^{\mu\nu} = 0, \tag{54}$$

$$\left[i\gamma^\mu\nabla_\mu - \frac{1}{8}F_{\alpha\beta}\left(i\gamma^{[\alpha}\gamma^{\beta]}\right) - \frac{1}{2}l_0^2\left(i\gamma^5\gamma^\mu\right)\left(i\bar{\psi}\gamma^5\gamma_\mu\right) - m\right]\psi = 0, \tag{55}$$

$$H^{\mu\nu} = F^{\mu\nu} + f^{\mu\nu};\ f^{\mu\nu} = 4l_0^2\left(i\bar{\psi}\gamma^{[\mu}\gamma^{\nu]}\psi\right), \nabla_\mu = \partial_\mu - \frac{1}{4}\omega_{ab\mu}\gamma^{[a}\gamma^{b]} \tag{56}$$

where ∇_μ is the 4D covariant derivative of the spinor field, $\omega_{ab\mu}$ is the 4D Ricci coefficients, a, b are the fier-bein indices, $E^{\mu\nu\alpha\beta}$ is the 4D absolutely antisymmetric tensor, l_0 is some length, [] means the antisymmetrization. This equation set is very complicated therefore we will consider here only the PSF + electrodynamics, *i.e.* gravity will be excluded.

Let us consider the case when we do not have the external electrical charges. It allows us to resolve the Maxwell equation (54)

$$F_{\mu\nu} = -f_{\mu\nu} = -4l_0^2 \left(i\bar{\psi}\gamma_{[\mu}\gamma_{\nu]}\psi\right). \tag{57}$$

Then the Dirac equation has the following form

$$\left\{i\gamma^\mu \nabla_\mu + \frac{l_0^2}{2}\left[\left(i\gamma^{[\mu}\gamma^{\nu]}\right)\left(i\bar{\psi}\gamma_{[\mu}\gamma_{\nu]}\psi\right) - \left(i\gamma^5\gamma^\mu\right)\left(i\bar{\psi}\gamma^5\gamma_\mu\psi\right)\right] - m\right\}\psi = 0. \tag{58}$$

Immediately we see that it is one of the variants of the non-linear Heisenberg equations (the Dirac equation + a non-linear term). The solution we will search in the following form

$$\psi(r,\theta,\varphi) = e^{i\omega t}\begin{pmatrix} f(r) \\ 0 \\ ig(r)\cos\theta \\ ig(r)\sin\theta e^{i\varphi}\end{pmatrix}. \tag{59}$$

After substitution in Eq. (58) we have

$$g' + \frac{2}{r}g + f(m+\omega) - l_0^2 f\left(f^2 - g^2\right) = 0, \tag{60}$$

$$f' + g\left(m-\omega\right) - l_0^2 g\left(f^2 - g^2\right) = 0. \tag{61}$$

In the Ref. [9] it is shown that such equations set has a regular solutions. At the origin $r = 0$ this solution can be expanded into a series

$$f(r) = f_0 + \frac{f_2}{2}r^2 + \ldots, \tag{62}$$

$$g(r) = g_1 r + \frac{g_3}{3!}r^3 + \ldots \tag{63}$$

with

$$g_1 = \frac{1}{3}f_*\left[l_0^2 f_*^2 - (m+\omega)\right], \tag{64}$$

$$f_2 = g_1\left[l_0^2 f_*^2 - (m-\omega)\right] \tag{65}$$

where f_* is a value of f_0 for which there is the above-mentioned regular solution for $f(r)$ and $g(r)$, *i.e.* for the another $f_0 \neq f_*$ the solution has an infinite energy.

The asymptotical behavior of the regular solution is

$$f(r) = \frac{\tilde{f}}{l_0\sqrt{m}}\frac{e^{-r\sqrt{m^2-\omega^2}}}{r} + \ldots, \tag{66}$$

$$g(r) = \frac{\tilde{f}}{l_0\sqrt{m}}\sqrt{\frac{m+\omega}{m-\omega}}\frac{e^{-r\sqrt{m^2-\omega^2}}}{r} + \ldots \quad . \tag{67}$$

where $\tilde{f}$ is some constant. Let us come back to Maxwell field. For our spinor ansatz we have the following components of the electric and magnetic fields

$$E^r(r) = -f^{tr}(r) = 8l_0^2 f(r)g(r), \tag{68}$$

$$H_\theta(r) = \frac{1}{2}\epsilon_{\theta r\varphi}f^{r\varphi}(r) = 4l_0^2\sin\theta\Big(f^2(r) + g^2(r)\Big), \tag{69}$$

$$H_r(r) = \frac{1}{2}r\theta\varphi\sqrt{-\gamma}f^{\theta\varphi}(r) = 4l_0^2\cos\theta\Big(g^2(r) - f^2(r)\Big). \tag{70}$$

Now we should do the following important simplifying remark. The ansatz for the spinor field (59) has a preferred direction (the spin direction) but the PSF is a quantum fluctuating object therefore we should average our results over the spin direction. It means that we should average electric and magnetic fields over θ that give us

$$H_\theta = H_r = 0, \tag{71}$$
$$E_r = 8l_0^2 f(r)g(r). \tag{72}$$

Now we are ready to formulate the result. Our solution with ψ and E_r describes a ball of the polarized spacetime foam filled with the electric field. Another words, the PSF can confine electric field in some volume and it is ***a pure quantum gravity phenomenon***. We can expect that the energy density of the electric field will be very high and it is interesting to compare the properties of such ball with the electric field (frozen in the PSF) with the Gamma Ray Burst (GRB) characteristics.

A. The comparison with the GRB

The basic purpose of this section is to show that the PSF ball with the confined electric field can have the same energy as the GRB and the linear sizes will not exceed the sizes of the GRB. At first we would like to write Eq's (60) (61) in the dimensionless form

$$\bar g' + \frac{2}{x}\bar g + \bar f(1-\beta) - \bar f\left(\bar f^2 - \bar g^2\right) = 0, \tag{73}$$
$$\bar f' + \bar g(1+\beta) - \bar g\left(\bar f^2 - \bar g^2\right) = 0, \tag{74}$$
$$\bar f(x) = \frac{l_0}{\sqrt{m}} f(r), \qquad \bar g(x) = \frac{l_0}{\sqrt{m}} g(r), \qquad x = rm, \qquad \beta = -\frac{\omega}{m}. \tag{75}$$

The numerical solution of these equations presented on the Fig.2.

And now we can calculate the energy of the electric field

$$W = \frac{1}{8\pi}\frac{c^4}{k}\int_0^\infty E_r^2 4\pi r^2 dr = 32\frac{c^4}{k}\frac{1}{m}\int_0^\infty \bar f^2(x)\bar g^2(x)x^2 dx \tag{76}$$

where the numerical coefficient $\frac{c^4}{k}$ is necessary for the conversion the natural units to the CGS units. The numerical value of the integral is of the order of 1. Therefore we can estimate the value of m

$$32\frac{c^4}{k}\frac{1}{m} \approx 10^{53} erg \tag{77}$$

where the energy of the GRB $W_{GRB} \approx 10^{53} erg$. Consequently

$$m \approx 4\times 10^{-3} cm^{-1} \qquad \text{and} \qquad r_0 \approx \frac{1}{\sqrt{m^2-\omega^2}} \approx 0.5\times 10^3 cm = 500m \tag{78}$$

where r_0 is the linear size of the ball. And finally we will estimate the value of the electric field

$$E_r = \frac{c^4}{k} 8 l_0^2 f g = \frac{8c^4}{k} m \bar f \bar g \tag{79}$$

Inside of the ball the values of the $\bar f$ and $\bar g$ are of the order of unity. It gives us

$$E_r \approx 10^{23} CGS \tag{80}$$

Consequently the PSF ball filled with the electric field can contain the energy of the order of the energy of the GRB and the linear sizes of this ball are of the order of $\approx 100 \div 1000m$. It is not surprising because it is a quantum gravity effect. The big question here is : can be such energy of the electric field extracted from the frozen state in the polarized spacetime foam ? Another questions are : how can be such objects created and how fast it will disintegrate ? Another interesting peculiarity of the spinor equation is that l_0 can be the Planck length $l_0 = l_{Pl}$ and nevertheless this microscopical equation can give the macroscopical energy and linear sizes !

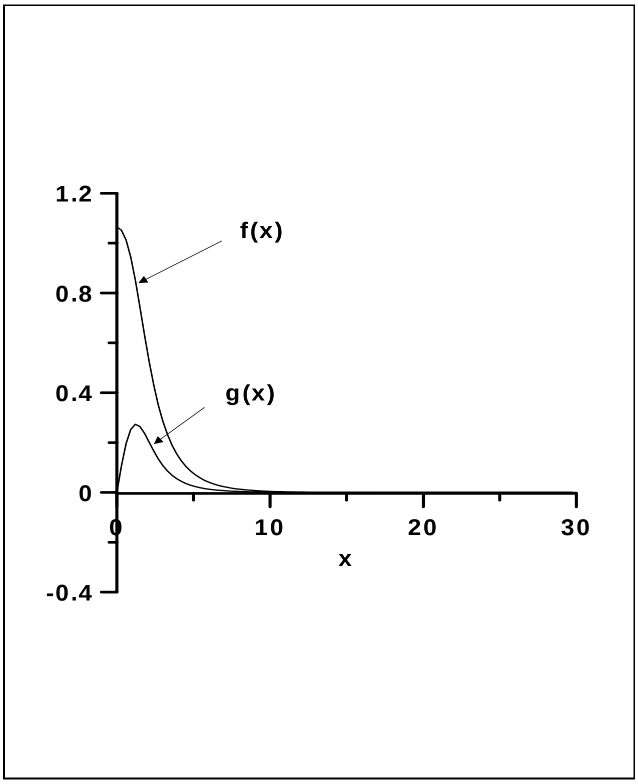

FIG. 2: The functions $\bar{f}(x)$ and $\bar{g}(x)$. $\beta = 0.9$, $f_* \approx 1.06477126$.

B. Non-linear Heisenberg equation

It is interesting to note that Eq. (58) is one of the variants of the Heisenberg equation for a non-linear spinor field [10]. This remark allows us to take a new interpretation of the solutions obtained in these Ref's : it is the PSF ball on a microscopical level.

VI. CONCLUSIONS

In this paper we offer an approximate model of the polarized spacetime foam. The microscopically description is based on the operator $W(x, y)$ of creating/destroying the minimalist wormhole. The minimalist wormhole is some approximation for the quantum handle when it is contracted into a point. The model for the quantum handle is given by the wormhole-like solution of the 5D Kaluza-Klein gravity. This metric contains off-diagonal components of the 5D metric G_{5t} and $G_{5\varphi}$ which can be considered as an electric and magnetic fields. It allows us to consider each quantum handle (minimalist wormhole) as an electric dipole. Obviously that in such point of view the spacetime foam can be polarized in the presence of an external electric field.

The operator $W(x, y)$ can be connected with either a scalar or a spinor field. Here the question arises : is this operator a consequence of quantum gravity or it is an additional stuff in quantum gravity describing the topology changes ? We assume that these fields can be dynamical. As a model of the scalar field we propose the dilaton field. In this case the dilaton field describes a polarization of the spacetime foam. In the result we have a renormalization of the bare electric charge and the finiteness of the energy of the electric field.

In the another variant a spinor field describes the polarization of the spacetime foam. We have considered the simplest case without external electric charges and gravity. We have shown that the polarized spacetime foam can confine an electric field in some finite region of the space. The magnitude of the confined electric field can be so much that the energy of this object can be compared with the energy of the Gamma Ray Burst and the linear sizes of this region do not exceed the sizes of the Gamma Ray Burst. It is necessary to note that in the consequences of the very high energy density of the electric field confined in the PSF the experimental verification of the effects connected with quantum gravity is very dangerous, much more dangerous then the nuclear bomb !

Finally we would like to summarize

- Each quantum handle in the spacetime foam is like to an electric dipole.
- It is possible to introduce an operator describing a quantum handle.
- In the presence of an external electric field the spacetime foam can be polarized.
- The polarized spacetime foam can renormalize a bare electric charge.
- The energy of the renormalized charge become finite.
- The polarized spacetime foam can can confine electric field into finite region.
- The energy of confined electric field can be compared with the energy of the Gamma Ray Burst.

The problems originating here are the following

- At the center of the renormalized charge there is a gravitational singularity. It is possible to avoid it in some more realistic scenario ?
- Must be quantized the scalar and spinor fields ? It is not trivial question because these fields are some approximate description of the spacetime foam and consequently they already connected with quantum gravity.
- How can be created the ball with the confined electric fields : is it a quantum fluctuation of the metric or it was created in the Early Universe and now only disintegrate ?
- What is the duration of life of these objects : exist they infinitely or disintegrate after some finite time ?
- What give us the switch-on the gravity : the ball will be the same with some modifications or something different ?

VII. ACKNOWLEDGMENT

I am very grateful for Viktor Gurovich for the fruitful discussion.

[1] J. Wheeler, Ann. of Phys., **2**, 604(1957).
[2] L. Smolin, *"Fermions and topology"*, gr-qc/9404010.
[3] V. Dzhunushaliev, *"Wormhole with Quantum Throat"*, gr-qc/0005008, to be published in Grav. Cosmol.; *"An Approximate Model of the Spacetime Foam"*, gr-qc/0006016.
[4] V. Dzhunushaliev, Mod. Phys. Lett. A **13**, 2179 (1998).
[5] V. Dzhunushaliev and D. Singleton, Phys. Rev. D **59**, 064018 (1999).
[6] K. Bronnikov, Int.J.Mod.Phys. **D4**, 491(1995), Grav. Cosmol., **1**, 67(1995).
[7] V .Dzhunushaliev, *"A geometrical interpretation of Grassmanian Coordinates"*, hep-th/0104129.
[8] Ruth Gregory and Jeffrey A. Harvey, Phys. Rev., **D47**, 2411, (1993).
[9] R. Finkelstein, R. LeLevier, M. Ruderman, Phys.Rev., **83**, 326(1951); R. Finkelstein, C. Fronsdal, P. Kaus, Phys.Rev., **103**, 1571(1956).
[10] W. Heisenberg, Nachr. Akad. Wiss. Göttingen, N8, 111(1953); W. Heisenberg, Zs. Naturforsch., **9a**, 292(1954); W. Heisenberg, F. Kortel und H. Mütter, Zs. Naturforsch., **10a**, 425(1955); W. Heisenberg, Zs. für Phys., **144**, 1(1956); P. Askali and W. Heisenberg, Zs. Naturforsch., **12a**, 177(1957); W. Heisenberg, Nucl. Phys., **4**, 532(1957); W. Heisenberg, Rev. Mod. Phys., **29**, 269(1957).